Introduction to Solid Modeling Parametric Modeling



Why draw 3D Models?

- 3D models are easier to interpret.
- Simulation under real-life conditions.
- Less expensive than building a physical model.
- 3D models can be used to perform finite element analysis (stress, deflection, thermal.....).
- 3D models can be used directly in manufacturing, Computer Numerical Control (CNC).
- Can be used for presentations and marketing.

3D Modeling

There are three basic types of three-dimensional computer geometric modeling methods:

- Wireframe modeling
- Surface modeling
- Solid modeling

- Contains information about the locations of all the points (vertices) and edges in space coordinates.
- Each vertex is defined by x, y, z coordinate.
- Edges are defined by a pair of vertices.
- Faces are defined as three or more edges.
- Wireframe is a collection of edges, there is no skin defining the area between the edges.



Advantages:

- Can quickly and efficiently convey information than multiview drawings.
- The only lines seen are the intersections of surfaces.
- Can be used for finite element analysis.
- Can be used as input for CNC machines to generate simple parts.
- Contain most of the information needed to create surface, solid and higher order models

Disadvantages:

- Do not represent an actual solids (no surface and volume).
- Cannot model complex curved surfaces.
- Cannot be used to calculate dynamic properties.
- Ambiguous views



• Uniqueness problem.



Surface Modeling

A surface model represents the skin of an object, these skins have no thickness or material type.

- Surface models define the surface features, as well as the edges, of objects.
- A mathematical function describes the path of a curve (parametric techniques).
- Surfaces are edited as single entities.

Surface Modeling

Advantages:

- Eliminates ambiguity and non-uniqueness present in wireframe models by hiding lines not seen.
- Renders the model for better visualization and presentation, objects appear more realistic.
- Provides the surface geometry for CNC machining.
- Provides the geometry needed for mold and die design.
- Can be used to design and analyze complex free-formed surfaces (ship hulls, airplane fuselages, car bodies, ...).
- Surface properties such as roughness, color and reflectivity can be assigned and demonstrated.

Surface Modeling

Disadvantages:

- Surface models provide no information about the inside of an object.
- Complicated computation, depending on the number of surfaces

Solid Models

In the solid modeling, the solid definitions include vertices (nodes), edges, surfaces, weight, and volume. The model is a complete and unambiguous representation of a precisely enclosed and filled volume.

Advantages:

- Has all the advantages of surface models (uniqueness, non-ambiguous, realistic, surface profile) plus volumetric information.
- Allows the designer to create multiple options for a design.
- 2D standard drawings, assembly drawing and exploded views are generated form the 3D model.

Solid Models

Advantages:

- Can easily be exported to different Finite Element Methods programs for analysis.
- Can be used in newly manufacturing techniques; computer integrated manufacturing (CIM), computer aided manufacturing (CAM) and design for manufacturability ans assembly (DFM, DFA)
- Mass and volumetric properties of an object can be easily obtained; total mass, mass center, area and mass moment of inertia, volume, radius of gyration, ...

• Volumetric and Mass properties of an object can be easily obtained.

Volume:	$V = \iiint_V dV$
Centroid:	$x_c = \frac{1}{V} \iiint_V x dV$
	$y_c = \frac{1}{V} \iiint_V y dV$
	$z_c = \frac{1}{V} \iiint_V z dV$
Moments of inertia:	$I_{xx} = \iiint_V (y^2 + z^2) dV$
2 2	$I_{yy} = \iiint_V (x^2 + z^2) dV$
	$I_{zz} = \iiint_V (y^2 + x^2) dV$
Products of inertia:	$I_{xy} = \iiint_V xy dV$
	$I_{yz} = \iiint_V yz dV$
	$I_{zx} = \iiint_V zx dV$

Corresponding mass properties are obtained if density is included.

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Disadvantages:

- More intensive computation than wireframe and surface modeling.
- Requires more powerful computers (faster with more memory and good graphics), not a problem any more.

Definition of a Solid Model

A solid model of an object is a more complete representation than its surface (wireframe) model. It provides more topological information in addition to the geometrical information which helps to represent the solid uniquely.



Geometry Vs Topology

Geometry:

Metrics and dimensions of the solid object. Location of the object in a chosen coordinate system

Topology:

Combinatorial information like connectivity, associativity and neighborhood information. Invisible relationship information.



History of Geometric Modeling

Year	Modeler	Developer
1972	PAP, PADL-I, PADL2	Univ. of Rochester, Voelcker & Requicha
1973	Build-I Build-II	Braid's CAD Group in Cambridge, UK
1973	TIPS-I	Hokkaido University, Japan
1975	GLIDE-I	Eastman's Group in CMU, USA
1975	Euler Ops Winged Edge, B-rep	Baumgart, Stanford Univ., USA
1981	Romulus	Evans and Sutherland, 1st commercial

Properties of Solid Models

- Rigidity: Shape of the solid is invariant w.r.t location/orientation
- Homogeneous 3-dimensionality: The solid boundaries must be in contact with the interior. No isolated and dangling edges are permitted.



Properties of Solid Models

- Finiteness and finite describability: Size is finite and a finite amount of information can describe the solid model.
- Closure under rigid motion and regularised Boolean operations: Movement and Boolean operations should produce other valid solids.
- Boundary determinism: The boundary must contain the solid and hence determine the solid distinctively.
- Any valid solid must be bounded, closed regular and semi-analytic subsets of E³

Major Modeling Schemes

- 1. Half Spaces.
- 2. Boundary Representation (B-Rep)
- 3. Constructive Solid Geometry (CSG)
- 4. Sweeping
- 5. Analytical Solid Modeling (ASM)
- 6. Cell decomposition
- 7. Spatial Ennumeration
- 8. Octree encoding
- 9. Primitive Instancing

Conversion Among Representations

Some representations can be converted to other represenations.

CSG to B-REP (but not B-rep to CSG)



Some Solid Modelers in Practice

Modeler	Developer	Primary Scheme	User Input
CATIA	IBM	CSG	BREP+CSG
GEOMOD / I-DEAS	SDRC/EDS	BREP	BREP+CSG
PATRAN-G	PDA ENGG.	ASM	HYPERPATCHES+CSG
PADL-2	CORNELL UNI.	CSG	CSG
SOLIDESIGN	COMPUTER VISION	BREP	BREP+CSG
UNISOLIDS / UNIGRAPHICS	McDONELL DOUGLAS	CSG	BREP+CSG
PRO-E	PARAMETRIC	BREP	BREP+CSG
SOL. MOD. SYS	INTERGRAPH	BREP	BREP+CSG

Solid Modeling

Boundary Representation (B-rep)

- A solid model is formed by defining the surfaces that form its boundary (edges and surfaces)
- The face of a B-rep represents an oriented surface, there are two sides to the surface; solid side (inside) and void side (outside), unlike faces in a wireframe.
- B-rep model is created using Euler operation
- Many Finite Element Method (FEM) programs use this method. Allows the interior meshing of the volume to be more easily controlled.

B-Rep Data Structure



Fa	ce Table	Edg	e Table	Ver	tex Table
Face	Edges	Edge	Vertices	Vertex	Coordinates
F ₁	E ₁ , E ₅ , E ₆	E ₁	V ₁ , V ₂	\mathbf{V}_1	$x_{1}^{}, y_{1}^{}, z_{1}^{}$
F ₂	E ₂ , E ₆ , E ₇	E_2	V ₂ , V ₃	V_2	x ₂ , y ₂ , z ₂
F ₃	E ₃ , E ₇ , E ₈	E ₃	V ₃ , V ₄	V ₃	x ₃ , y ₃ , z ₃
F ₄	E ₄ , E ₈ , E ₅	E ₄	V ₄ , V ₁	V_4	x ₄ , y ₄ , z ₄
F ₅	E_1, E_2, E_3, E_4	E ₅	V ₁ , V ₅	V_5	x ₅ , y ₅ , z ₅
		E ₆	V ₂ , V ₅	V ₆	x ₆ , y ₆ , z ₆
		E ₇	V ₃ , V ₅		
		E ₈	V ₄ , V ₅		
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Euler Operators

Geometric entities stored in B-Rep data structures are the *shell, face, loop, edge,* and *vertex.*

Operators are needed to manipulate these entities (e.g., an operator to make an edge, an operator to delete an edge,...)

Boundary Representaion (B-rep)

- Closed Surface: One that is continuous without breaks.
- Orientable Surface :One in which it is possible to distinguish two sides by using surface normals to point to the inside or outside of the solid under consideration.
- Boundary Model :Boundary model of an object is comprised of closed and orientable faces, edges and vertices. A database of a boundary model contains both its topology and geometry.
- Topology :Created by Euler operations
- Geometry :Includes coordinates of vertices, rigid motions and transformations

Boundary Representaion (B-rep)

- Involves surfaces that are
 - closed, oriented manifolds embedded in 3-space
- A manifold surface:
 - each point is homeomorphic to a disc
- A manifold surface is oriented if:
 - any path on the manifold maintains the orientation of the normal
- An oriented manifold surface is closed if:
 - it partitions 3-space into points inside, on, and outside the surface
- A closed, oriented manifold is embedded in 3space if:
 - Geometric (and not just topological) information is known

Boundary Representaion (B-rep)

 Non-manifold surfaces







Figure 3.35 A Nonmultiki Object

Figure 2.29 Two Possible Topologics

 Non-oriented Manifolds



Moebius strip



Klein bottle

Object Modeling with B-rep

Both polyhedra and curved objects can be modeled using the following primitives

- Vertex: A unique point (ordered triplet) in space.
- Edge :A finite, non-selfintersecting directed space curve bounded by two vertices that are not necessarily distinct.
- Face :Finite, connected, non-selfintersecting region of a closed, oriented surface bounded by one or more loops.
- Loop :An ordered alternating sequence of vertices and edges. A loop defines non-selfintersecting piecewise closed space curve which may be a boundary of a face.
- Body :An independent solid.
- Genus :Hole or handle.

Boundary Representation

E = 4

V = 4

F = 1



Modified objects

Nonsense object

 Euler Operations (Euler – Poincare' Law): The validity of resulting solids is ensured via Euler operations which can be built into CAD/CAM systems.

Leonhard Euler (1707 – 1783) Henri Poincaré (1854 – 1912)



Euler-Poincare Law

- Euler (1752) a Swiss mathematician proved that polyhedra that are homomorphic to a sphere are topologically valid if they satisfy the equation:
- F E + V L = 2(B G)GeneralF E + V = 2Simple SolidsF E + V L = B GOpen ObjectsF=FaceE=EdgeV=VerticesB=BodiesL=Faces' inner
LoopG=Genus

Name	lmage	Vertices V	Edges <i>E</i>	Faces <i>F</i>	Euler characteristic: V – E + F
Tetrahedron		4	6	4	2
Hexahedron or cube		8	12	6	2
Octahedron		6	12	8	2
Dodecahedron		20	30	12	2
lcosahedron		12	30	20	2

Validity Checking for Simple Solids

F - E + V = 2 Simple Solids



Validity Checking for Simple Solids F - E + V = 2 Simple Solids



Loops (rings), Genus & Bodies

- Genus zero
- Genus one
- Genus two










Validity Checking for Polyhedra with inner loops

F - E + V - L = 2(B - G) General



$$E = 36$$

 $F = 16$
 $V = 24$
 $L = 2$
 $B = 1$
 $G = 0$
 $16 - 36 + 24 - 2 = 2(1 - 0) = 2$

Validity Checking for Polyhedra with holes

	F-	E+V-L	= 2(B-G)	General		
Interior I			E = 24			
			F =12			
			V = 16			
ğ			L = 0			
e ()			B = 2			
ò.			G=0			
9	-		12-24+16-0=	=2(2-0)=4		
			E = 24	F = 11		
			V = 16	L = 1		
			B = 1	G = 0		
			11-24+16	-1 = 2(1-0) = 2		
Surface hole						



Validity Checking for Open Objects F - E + V - L = B - G



Exact Vs Faceted B-rep Schemes

- Exact B-rep: If the curved objects are represented by way of equations of the underlying curves and sufraces, then the scheme is Exact B-rep.
- Approximate or faceted B-rep : In this scheme of boundary representation any curved face divided into planar faces. It is also know as tessellation representation.



Exact B-rep: Cylinder and Sphere



Faceted cylinder and sphere



Building Operations F - E + V - L = 2(B - G) General

The basis of the Euler operations is the above equation. M and K stand for Make and Kill respectively.

Operation	Operator	Complement	Description
Initiate Database and begin creation	MBFV	KBFV	Make Body Face Vertex
Create edges and	MEV	KEV	Make Edge Vertex
vertices	MEKL	KEML	Make Edge Kill Loop
Create edges and	MEF	KEF	Make Edge Face
taces	MEKBFL	KEMBFL	Make Edge Kill Body, Face Loop
	MFKLG	KFMLG	Make Edge Kill Loop Genus
Glue	KFEVMG	MFEVKG	Kill Face Edge Vertex Make Genus
	KFEVB	MFEVB	Kill Face Edge Vertex Body
Composite	MME	KME	Make Multiple Edges
Operations	ESPLIT	ESQUEEZE	Edge Split
	KVE		Kill Vertex Edge







Merits and Demerits of Euler Operations

If the operator acts on a valid topology and the state transition it generates is valid, then the resulting topology is a valid solid. Therefore, Euler's law is never verified explicitly by the modeling system.

- Merits:
 - They ensure creating valid topology
 - They provide full generality and reasonable simplicity
 - They achieve a higher semantic level than that of manipulating faces, edges and vertices directly

Demerits :

- They do not provide any geometrical information to define a solid polyhedron
- They do not impose any restriction on surface orientation, face planarity, or surface self intersection

Advantages and Disadvantages of B-rep

Advantages:

- It is historically a popular modeling scheme related closely to traditional drafting
- It is very appropriate tool to construct quite unusual shapes like aircraft fuselage and automobile bodies that are difficult to build using primitives
- It is relatively simple to convert a B-rep model into a wireframe model because its boundary definition is similar to the wireframe definitions
- In applications B-rep algorithms are reliable and competitive to CSG based algorithms

Disadvantages :

- It requires large storage space as it stores the explicit definitions of the model boundaries
- It is more verbose than CSG
- Faceted B-rep is not suitable for manufacturing applications

Constructive Solid Geometry, CSG

- CSG defines a model in terms of combining basic and generated (using extrusion and sweeping operation) solid shapes.
- CSG uses Boolean operations to construct a model (George Boole, 1815-1864, invented Boolean algebra).
- There are three basic Boolean operations:

<u>Union</u> (Unite, join) - the operation combines two volumes included in the different solids into a single solid.

Subtract (cut) - the operation subtracts the volume of one solid from the other solid object.

Intersection - the operation keeps only the volume common to both solids

Primitive Solids and Boolean Operations

The basic primitive solid:



Primitive Solids

The location of the insertion base or base point and default axes orientation.









Union



Implementing Boolean Operation

Consider solids A and B.



The intersection curves of all the faces of solid *A* and *B* are calculated. These intersections are inscribed on the associated faces of the two solids.



The faces of solid A are classified according to their relative location with respect to solid B. Each face is tested to determine whether it is located inside, outside, or on the boundary surface of solid B. The faces in group A_1 are outside solid B, and those of group B_1 are inside solid A.



Groups of faces are collected according to the specific Boolean operation and the unnecessary face groups are eliminated. For example, for union operation, group A_1 and B_2 are collected and A_2 and B_1 are eliminated.



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Solid Modeling Example Using CSG

Plan your modeling strategy before you start creating the solid model





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Creating Solid Models Parametric Modeling Concept

- *Parametric* is a term used to describe a dimension's ability to change the shape of model geometry if the dimension value is modified.
- *Feature-based* is a term used to describe the various components of a model. For example, a part can consists of various types of features such as holes, grooves, fillets, and chamfers.
- Parametric modeler are featured-based, parametric, solid modeling design program: SolidWorks, Pro-Engineer, Unigraphics (CSG and parametric),

- In parametric modeling, dimensions control the model.
- Design intent is how your model will react when dimension values are changed.

Example:

The drawing shows the intent of the designer that the inclined plane (chamfer) should have a flat area measuring 2.5 inches and that it should start at a point 1.25 inches from the base of the drawing. These parameters are what the designer deemed significant for this model.



Remember that the placement of dimensions is very important because they are being used to drive the shape of the geometry. If the 2.5 in. vertical dimension increases, the 2.5 in. flat across the chamfer will be maintained, but its angle will change.

In this drawing, what is important to the designer is the vertical location and horizontal dimension of the chamfer, rather than the flat of the chamfer.

In the last drawing, the designer calls for a specific angle for the chamfer. In this case the angle of the chamfer should be dimensioned.







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Design Notes

- Keep in mind that dimensioning scheme can be changed at any time. You are not locked into a specific design. You can also design without dimensioning, rough out a sketch, and then later go back and fully define it.
- Do not be concerned with dimensioning to datum or stacked tolerances in the part. Those issues can be addressed in the drawing layout. Be more concerned with your design intent.

The ability to go back on some earlier stage in the design process and make changes by editing a sketch or changing some dimensions is extremely important to a designer. This is the main advantage of a *parametric* (SolidWorks, Unigraphics, Inventor, Pro-Engineer) over a *non-parametric* modeler (AutoCAD 3D modeler – Boolean operation)

Example:

Let's assume that it is desired to design a part consisting of a ring with a certain thickness and a series of counterbore holes along the perimeter.





Boolean operation

Make the base part by creating two cylinders and subtract the small one from the large one



Create the solid geometry that will become the counterbore holes and generate the pattern.



Position the pattern about the perimeter of the base part. Locating the holes is critical to creating an accurate solid model.

Subtract the pattern from the base part to create the actual holes.





What would happen if you had to come back to this part to change the thickness of the ring or size of the counterbore holes?

Since Boolean operation was used to create the part, changing the thickness would not increase the height of the holes. There is no association between the thickness and the hole pattern location.

Parametric modeling (SolidWorks, ProE, UG, ...)

Create the initial base, the ring, by extruding the profile (circles) in a particular direction (Pro/E or SolidWorks) or use primitive solids and Boolean operation (UG).

Create the counterbore as a feature. Select the top surface of the ring and either sketch the two holes and extrude at different depth or use the hole feature option.





The next step would be to pattern the hole. The pattern would actually be considered a feature in itself, and would have its set of parametric variables, such as the number of copies and the angle between copies.



The model created would be identical to the one created using Boolean operation, **but with intelligence built into the model.**

The true power of parametric modeling shines through when *design changes* need to be made. The design modification is made by simply changing a dimension.



Since the counterbore is associated with the top surface of the ring, any changes in the thickness of the ring would automatically be reflected on the counterbore depth.

Sketching and Features

When discussing the mind-set needed for working with parametric modelers, there are two topics that need to be expanded: *Sketching and Features*

<u>Sketching</u>

- Take the word sketch literally. A sketch should be just that, a *sketch*.
- When sketching, it is not necessary to create geometry with accuracy. Lines, arcs, and additional geometry need not be created with exact dimensions in mind.
- When the dimensions are added, the sketch will change size and shape. This is the essence of Parametric Modeling.

In short, the sketch need only be the approximate size and shape of the part being designed. When dimensions and constraints are added, they will drive the size and the shape of the geometry.
Sketching and Features

<u>Features</u>

- Sketched Feature
 - Create a 2D sketch.
 - Create a feature from the sketch by extruding, revolving, sweeping, lofting and blending.



Revolved feature

Extruded feature

Creating Features from Sketches



Sketching and Features

- Applied Feature
 - > Applied feature does not require a sketch.
 - \succ They are applied directly to the model.
 - Fillets and chamfers are very common applied features.



Summary – Solid Modeling Methods

Primitive creation modeling

A solid model is created by retrieving primitive solids and performing Boolean operations

Sweeping function

Creates a solid by translating, revolving or sweeping a predefined 2D shape (*Sketching*). If geometric and dimensional constraints are imposed, it is called *Parametric Modeling*.

Feature-based Modeling

Models a solid by using familiar shapes; holes, slots, grooves, pockets, chamfers, fillets.....

Summary – Solid Modeling Methods

Modifying functions

Rounding (or blending) and lifting.

Boundary Modeling

Lower entities of a solid (vertices, edges and faces) are directly manipulated.

Sketching

- Take the word sketch literally. A sketch should be just that, a *sketch*.
- Sketches are able to capture the designer's intent for the part like no other technique.
- The sketch is the best tool for creating solids because of flexibility in shapes and the ability to edit.
- Ideal for creating unusual freeform shapes.

Sketching

- A sketch is a set of two dimensional curves joined in a string that when swept or extruded forms a solid body.
- When sketching it is not necessary to create geometry with accuracy.
- Lines, arcs, and additional geometry need not be created with exact dimensions in mind.

Sketching

- When the dimensions are added, the sketch will change size and shape. This is the essence of *parametric modeling*
- In short, the sketch need only be the approximate size and shape of the part being designed. The geometric constraints and dimensions, when added, will drive the size and the shape of the geometry.
- Curves are parametrically associated to each other and the solid that is created by them.